

# Solving Aristotle's Puzzle

Evolutionary Algorithms for Your Everyday  
Task

Marco Neumann

**BlueYonder**  
a jda. company

“Aristotle’s Puzzle” is officially referred to as “Magic Hexagon”!

**BlueYonder**  
a jda. company

**jda.**  
Plan to deliver™

# Selected Customers

BlueYonder  
a jda. company

**bon  
prix**

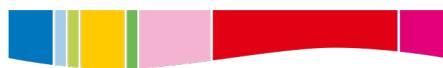
**dm**



**orsay**



**SELGROS**  
cash & carry



**Ernsting's family**  
Von fröhlichen Familien empfohlen.

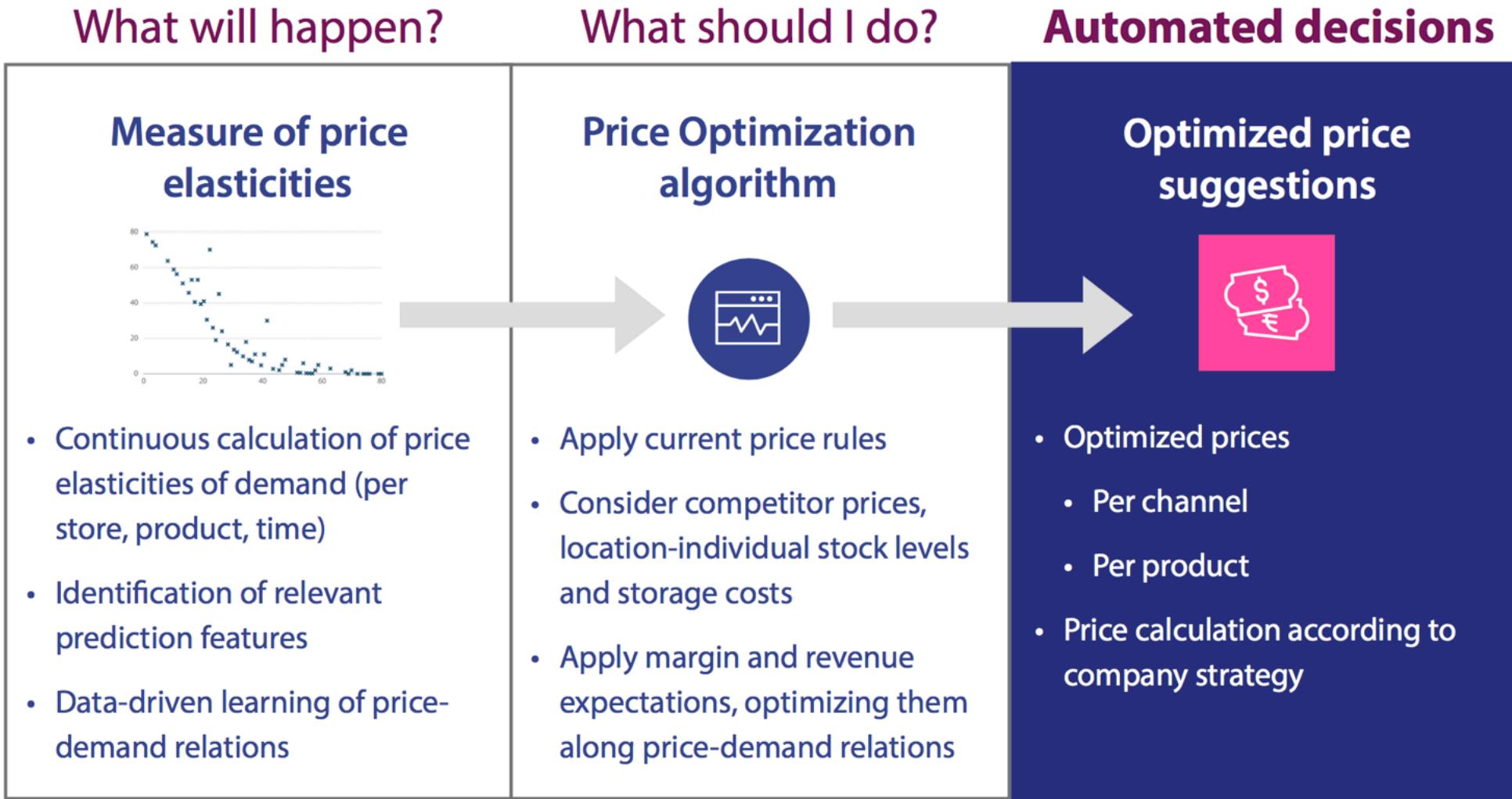
**GLOBUS**  
SAVOIR VIVRE



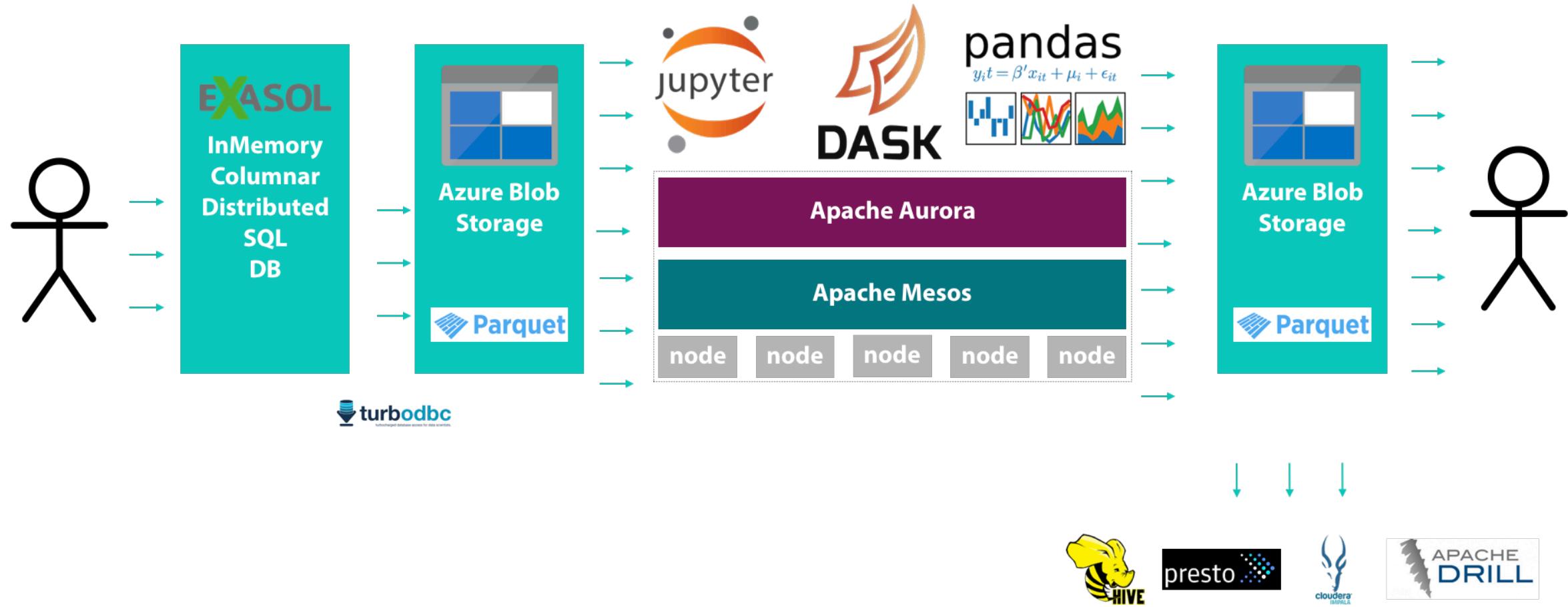
**Morrisons**  
Since 1899

**OTTO**

# Dynamic Price Optimization



# Data Flow for Machine Learning



# Want to join?

We are looking for open-minded candidates for our development teams:

**Data Scientist**

**Software Engineer**

**Data Engineer**

**Site Reliability Engineer**

## **Application via E-Mail:**

Contact Person: Inga Binder

Subject Line should contain “Meetup”

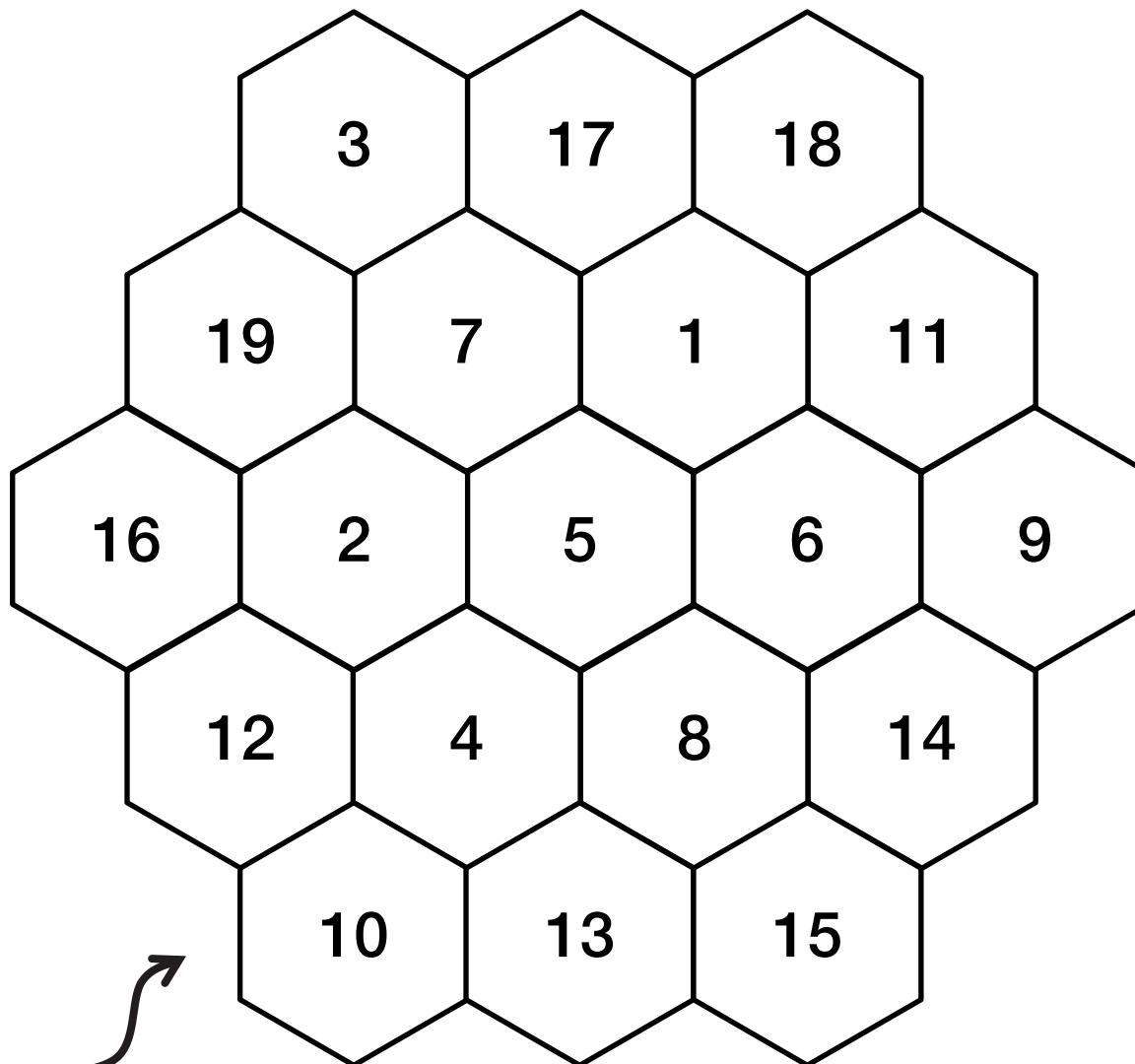
[PD-Candidates-GER@jda.com](mailto:PD-Candidates-GER@jda.com)

## **Job Listing:**

<https://stackoverflow.com/jobs/companies/blue-yonder-gmbh>

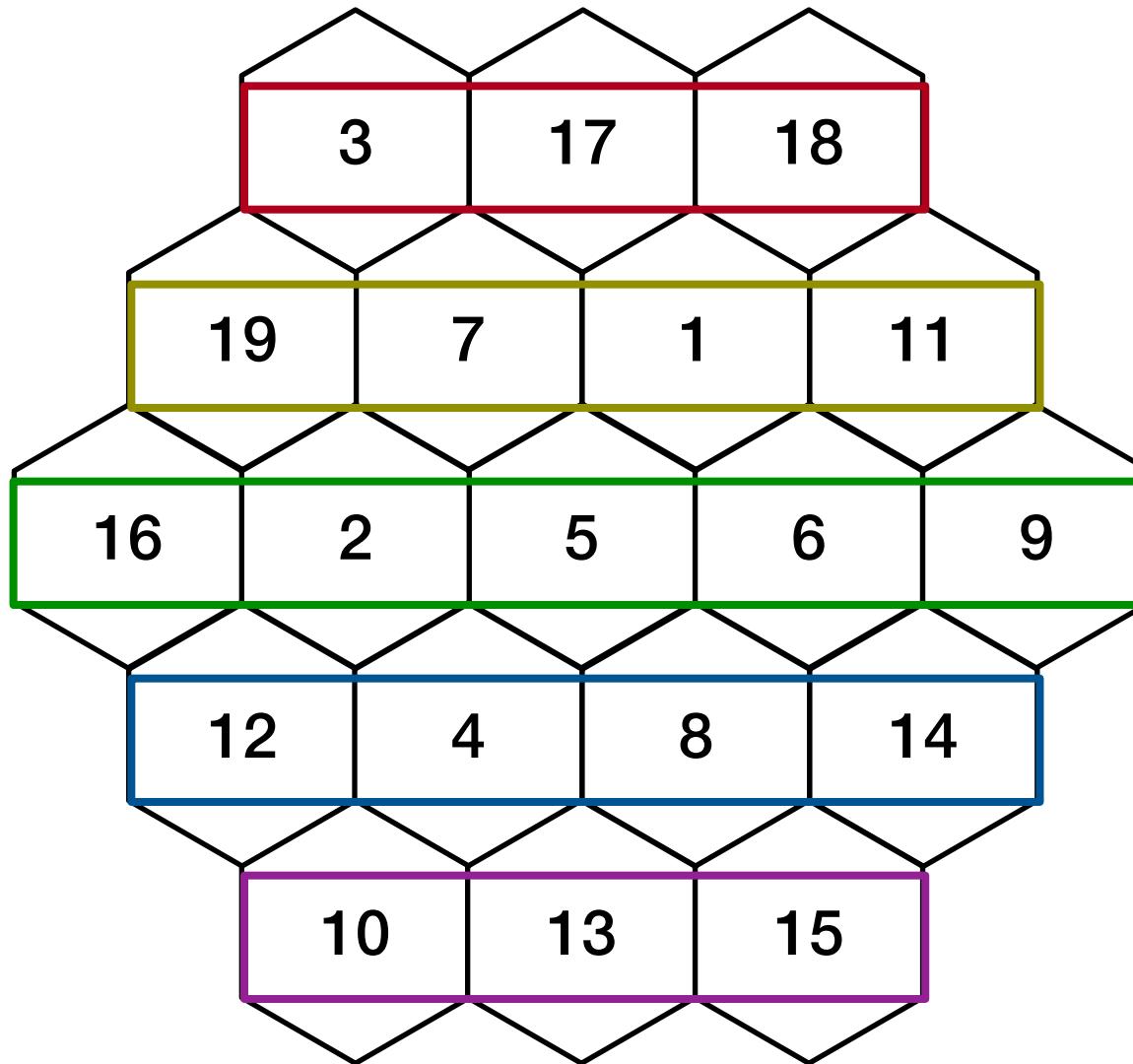
# The Problem

19 Tokens

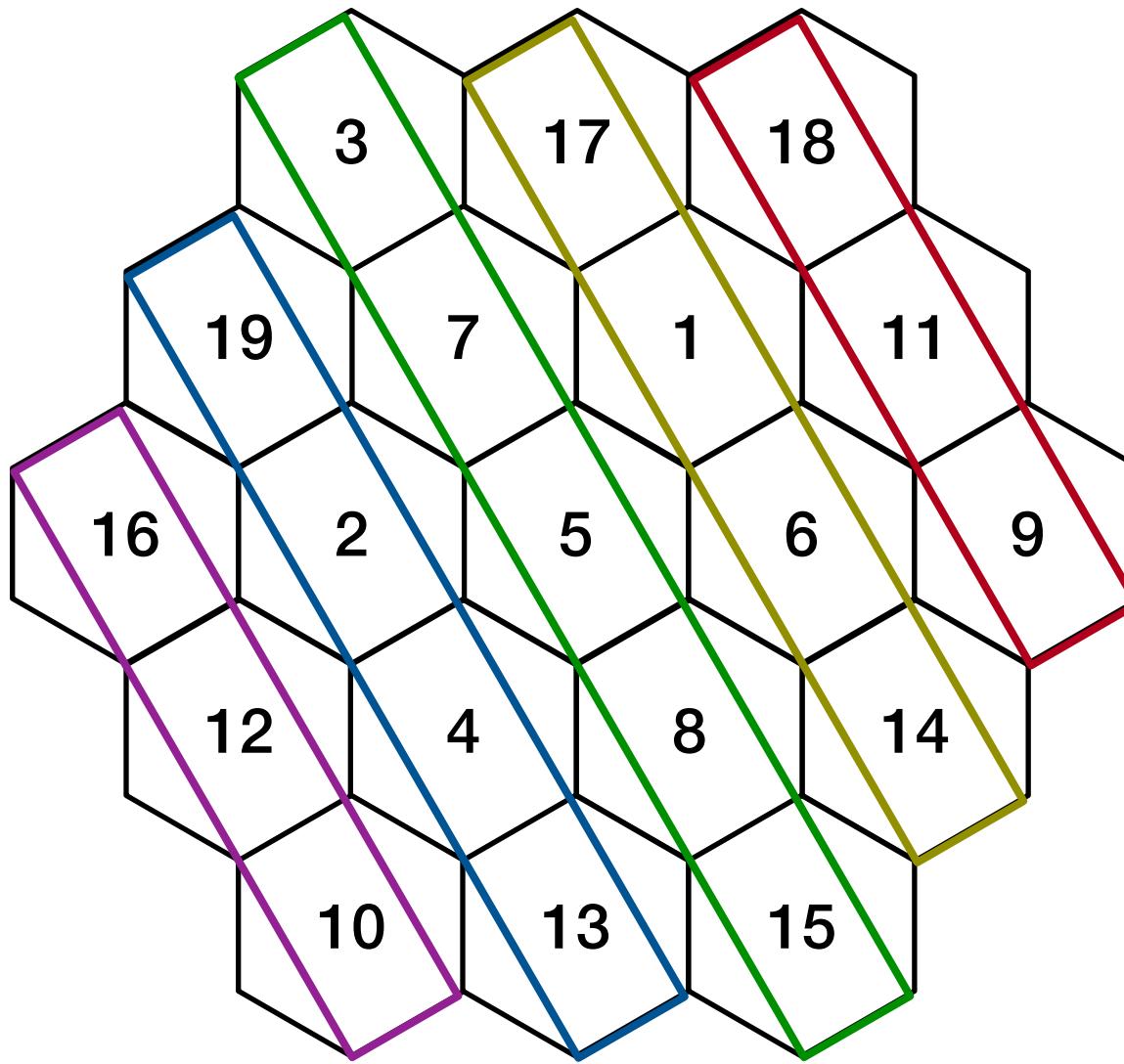


*That's actually the solution*

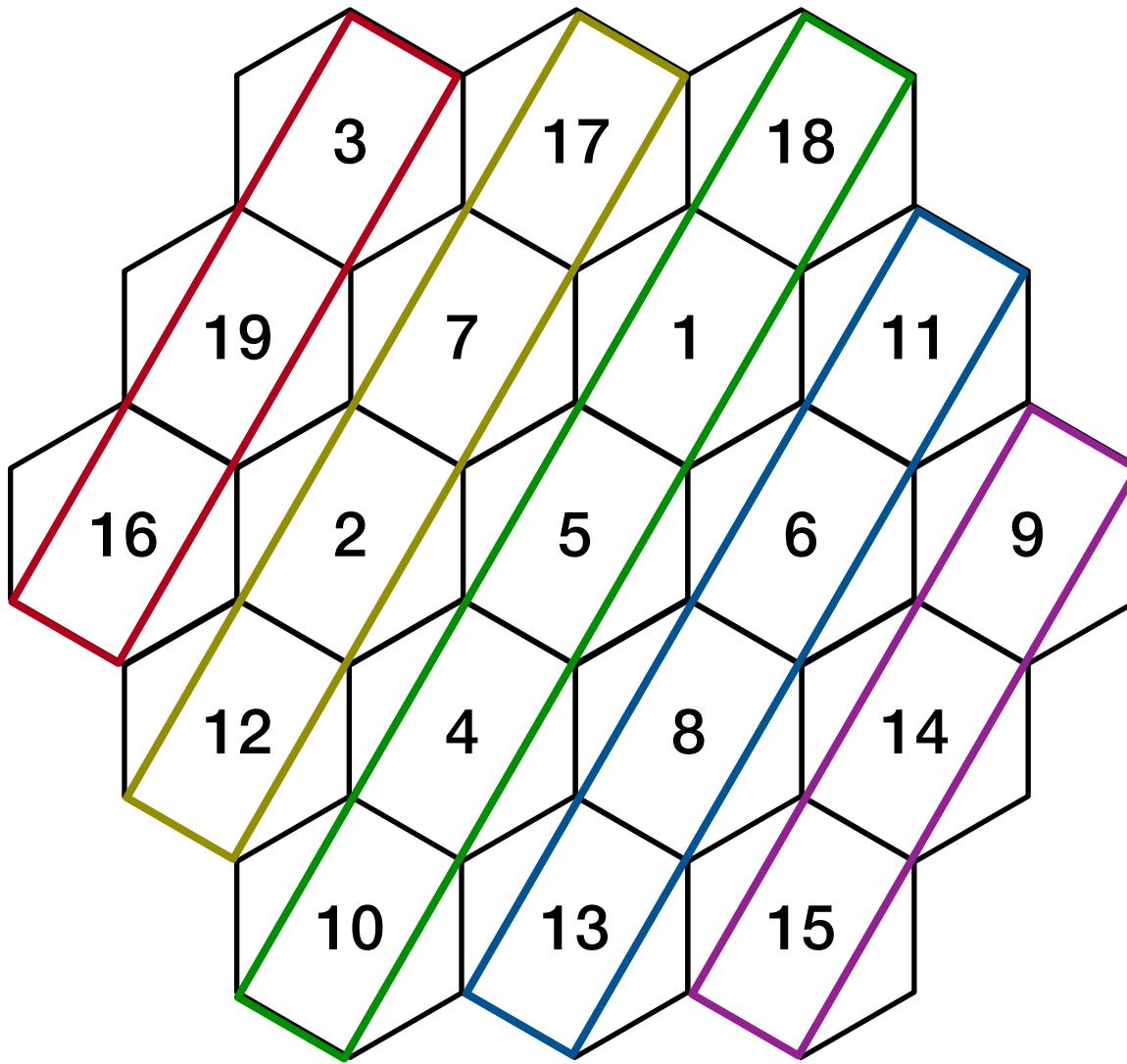
$$\sum_{i \in R} T_i = 38$$



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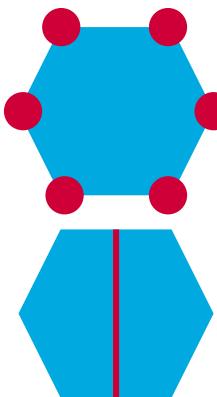


# The Solution

# Brute force

- ◆  $19! \approx 1.2 \times 10^{17}$  combinations ← *Age of the solar system:  $1.44 \times 10^{17} s$*
- ◆ Assume:
  - a processor core that can generate + check  $1 \times 10^9$  combinations per second
  - 100 cores
- ◆ Checking all takes  $\approx 1.2 \times 10^6 s \approx 338h \approx 14d$

- ◆ Assume 1 real solution with:
  - 6 rotations
  - 2 mirror images



- ◆ So on average we expect  $\approx \frac{14d}{6 \times 2 \times 2} \approx 14h$



# Evolutionary Algorithm

Generate



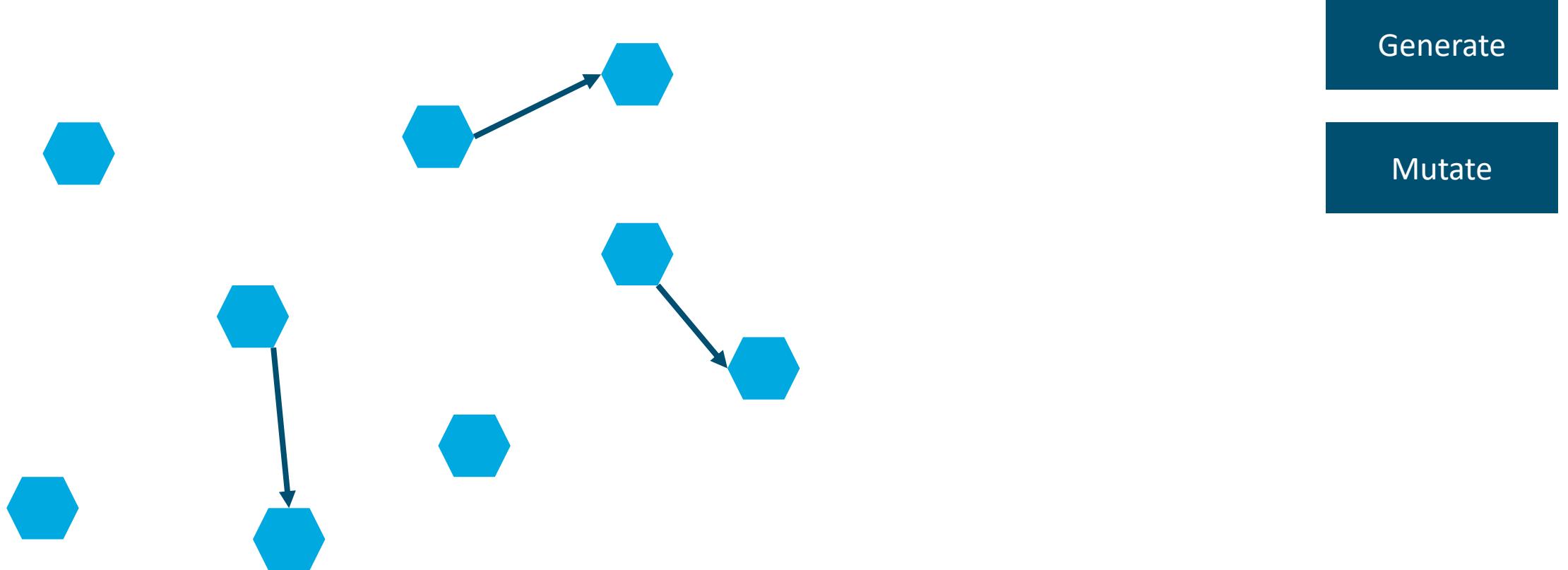
# Evolutionary Algorithm



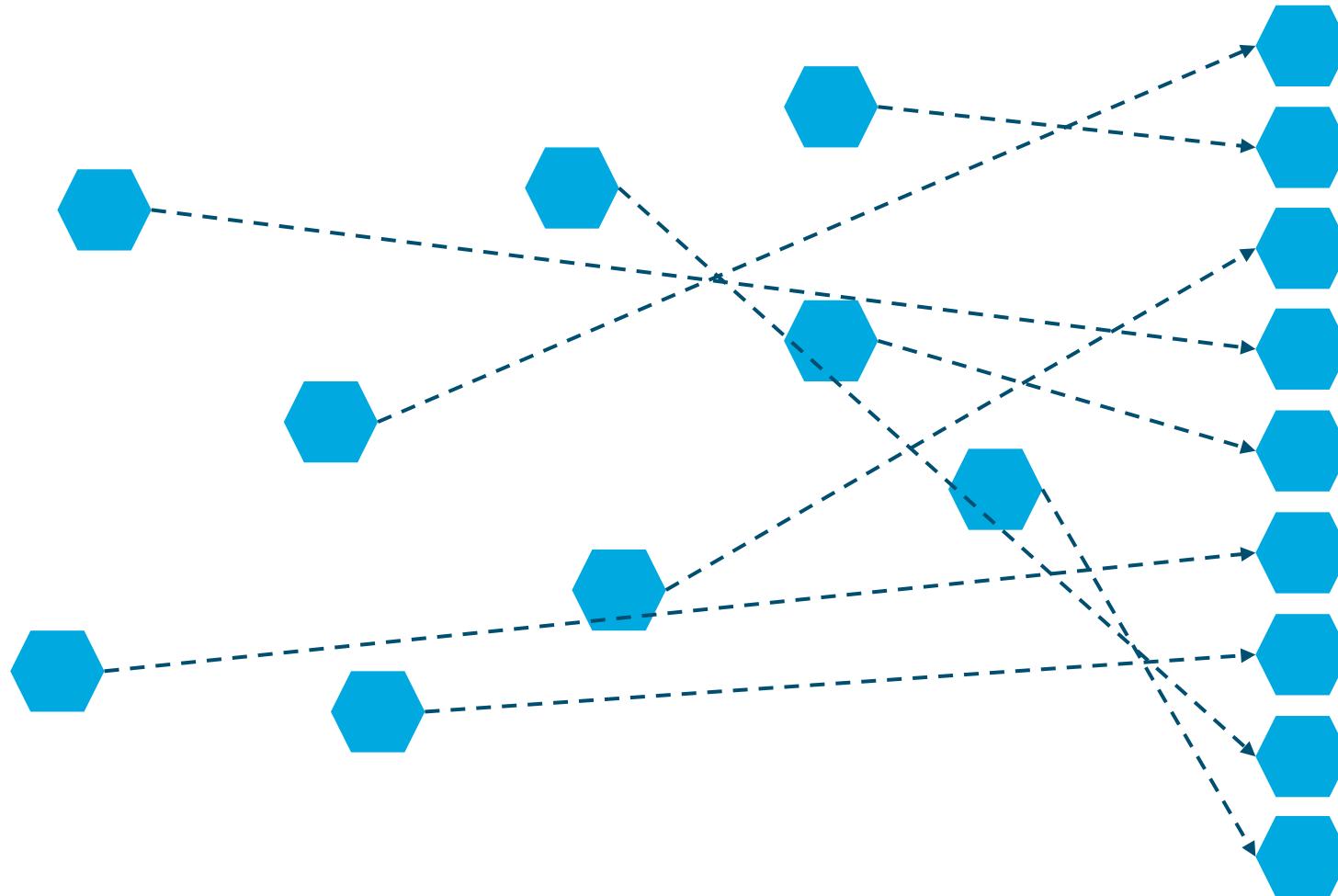
Generate

Mutate

# Evolutionary Algorithm



# Evolutionary Algorithm

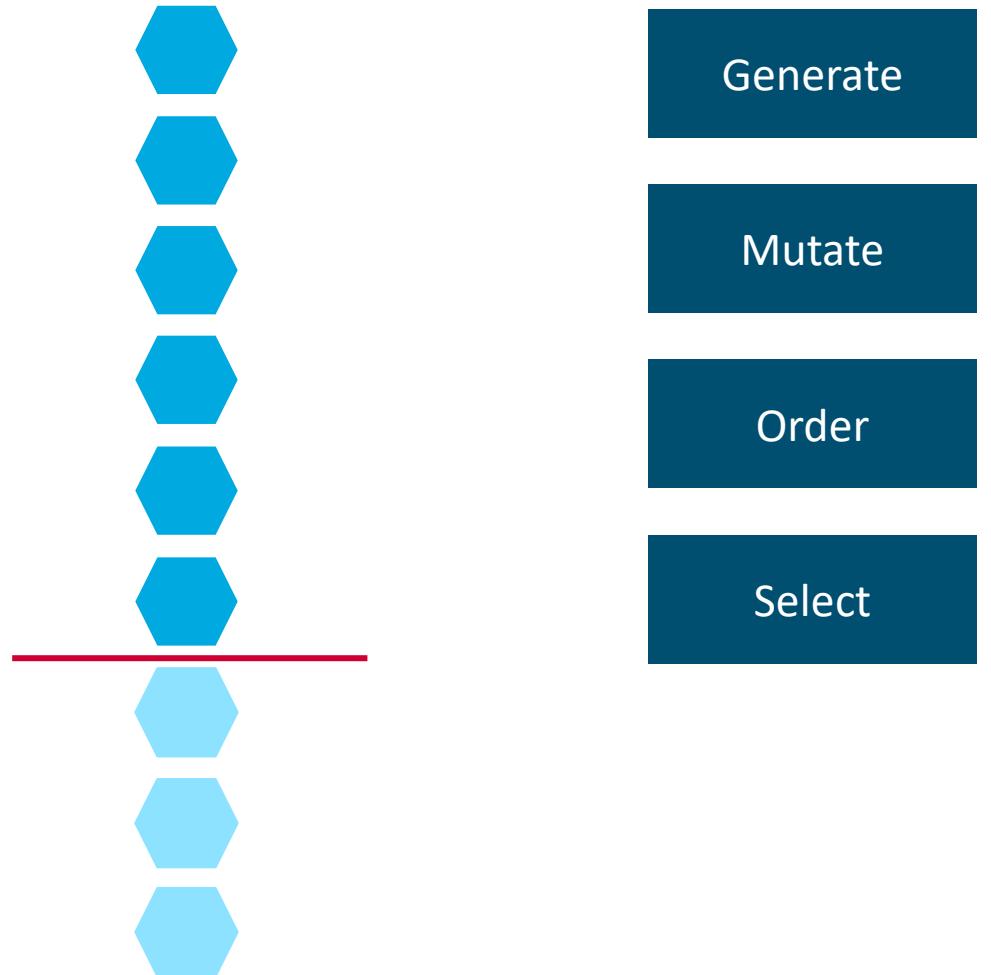


Generate

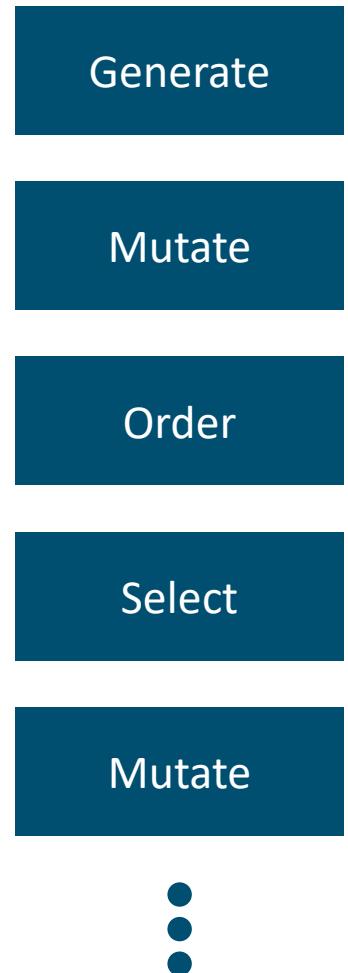
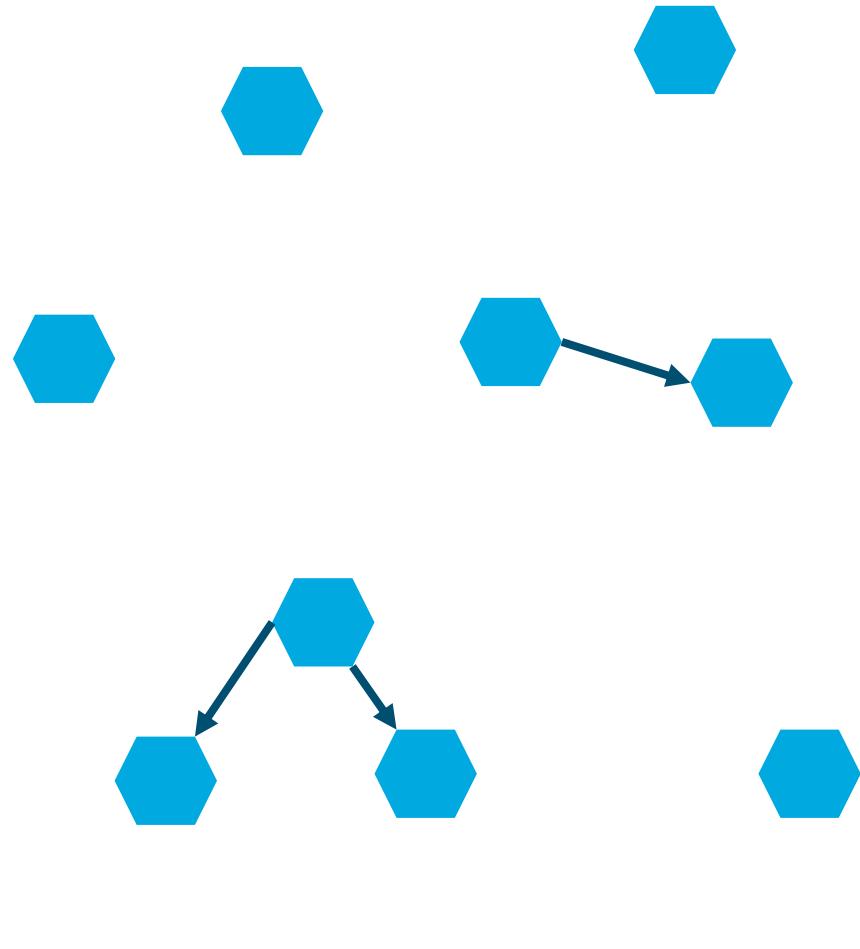
Mutate

Order

# Evolutionary Algorithm

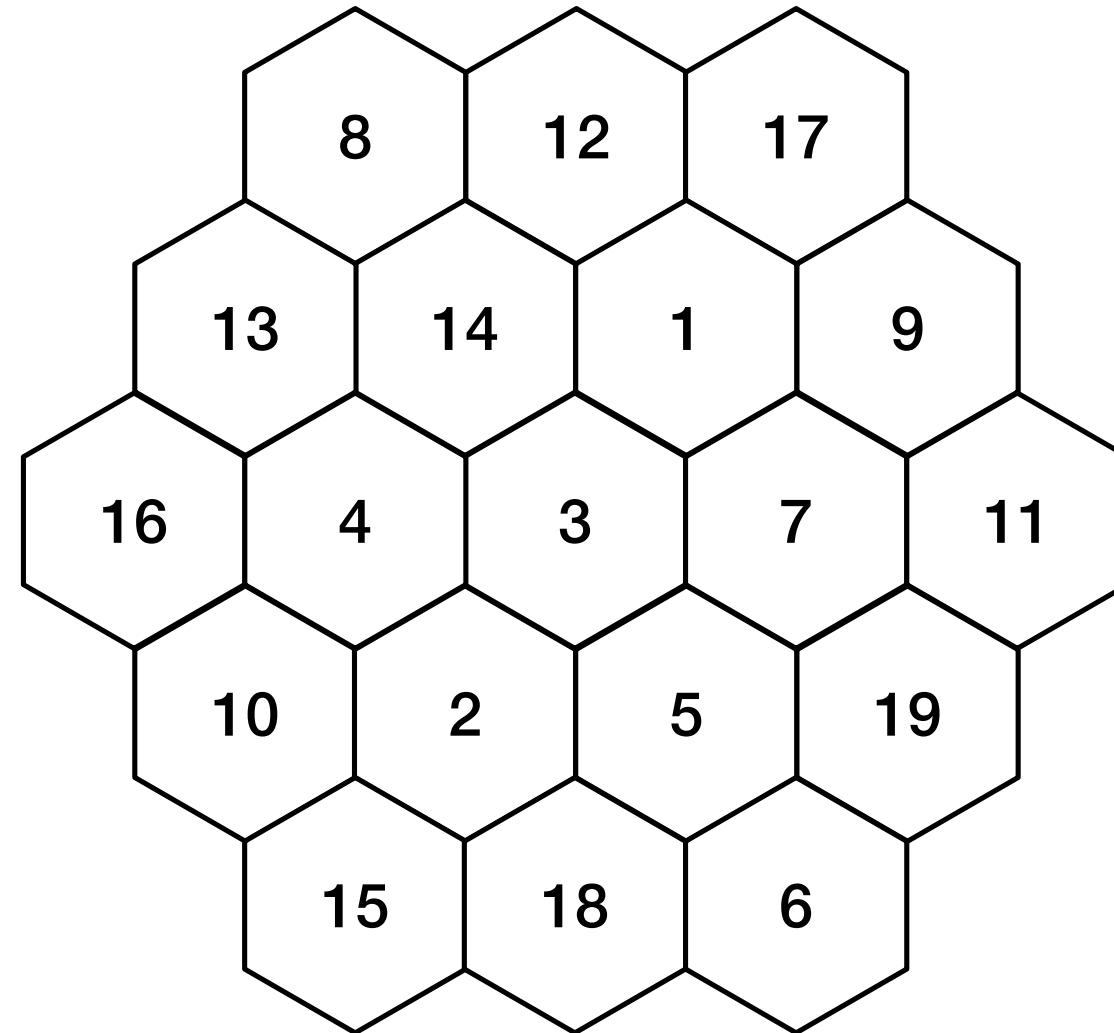


# Evolutionary Algorithm



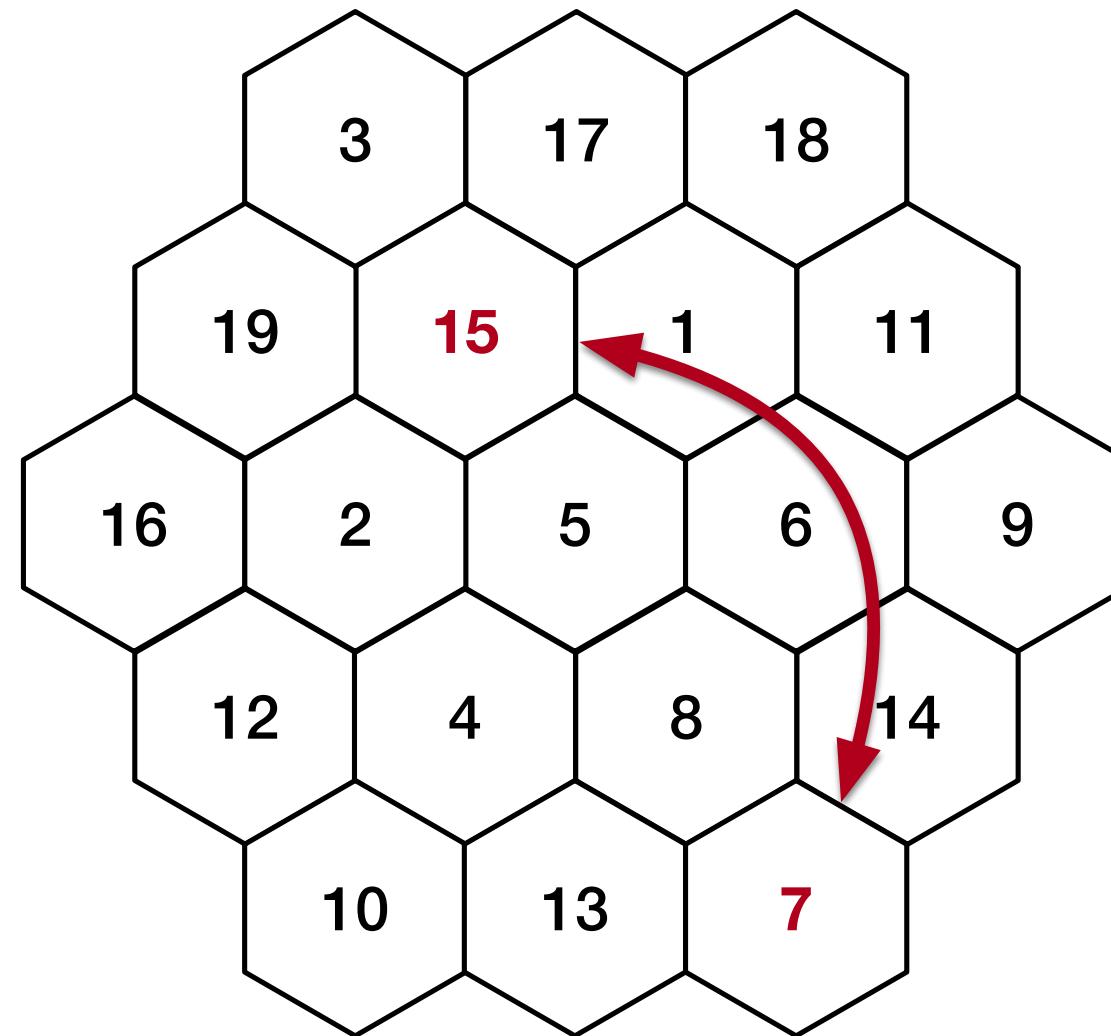
# Candidate Generation

Choose random permutation

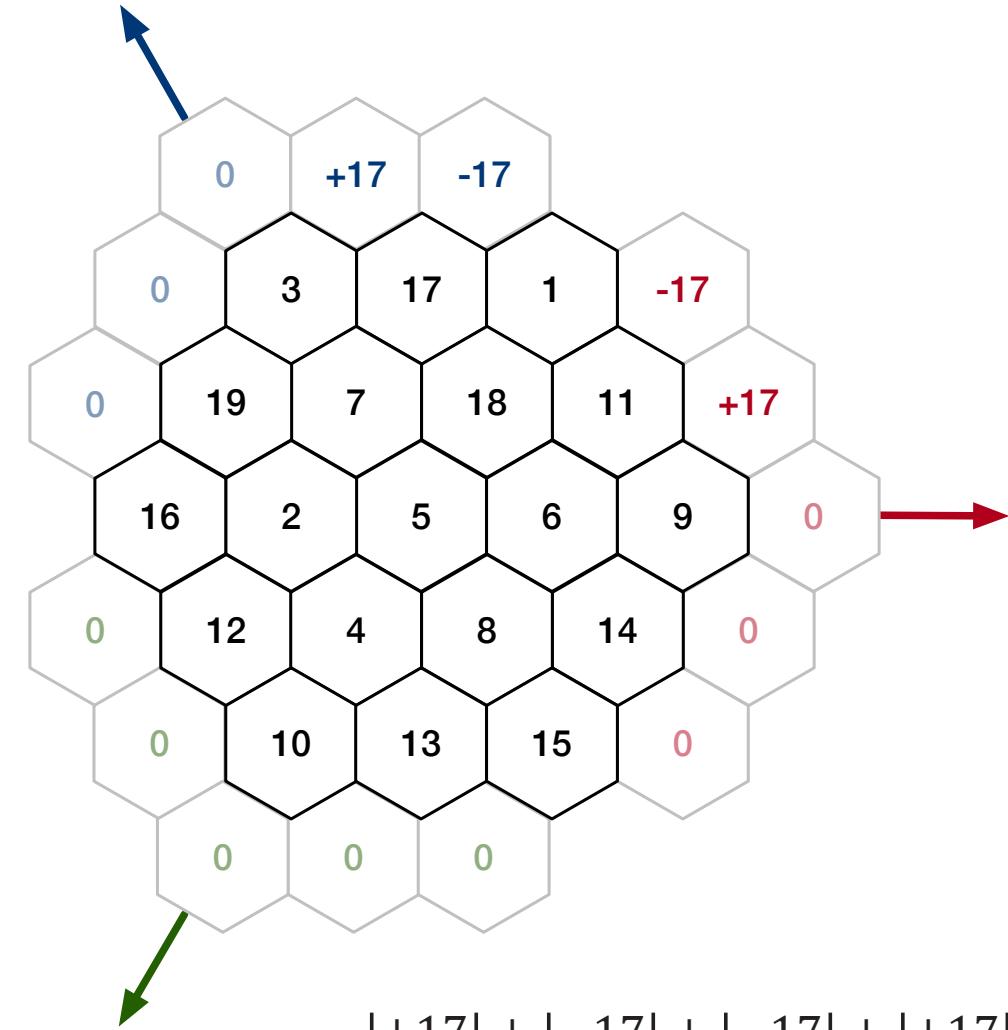
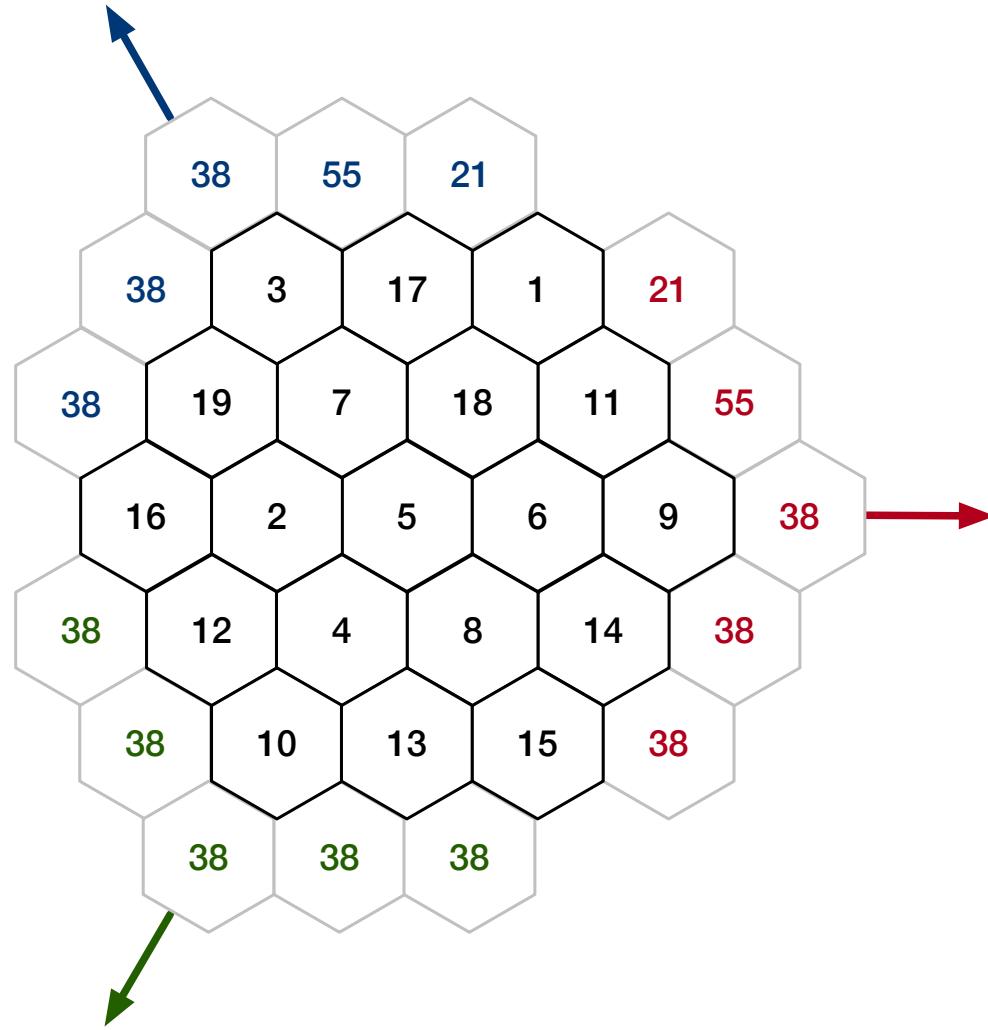


# Mutation

Swap random tokens



# Fitness Function



# Improvement #1

## **Issue:**

Fit candidates might get duplicated, take over population.

## **Solution:**

Keep unique candidates ([HashSet](#)).

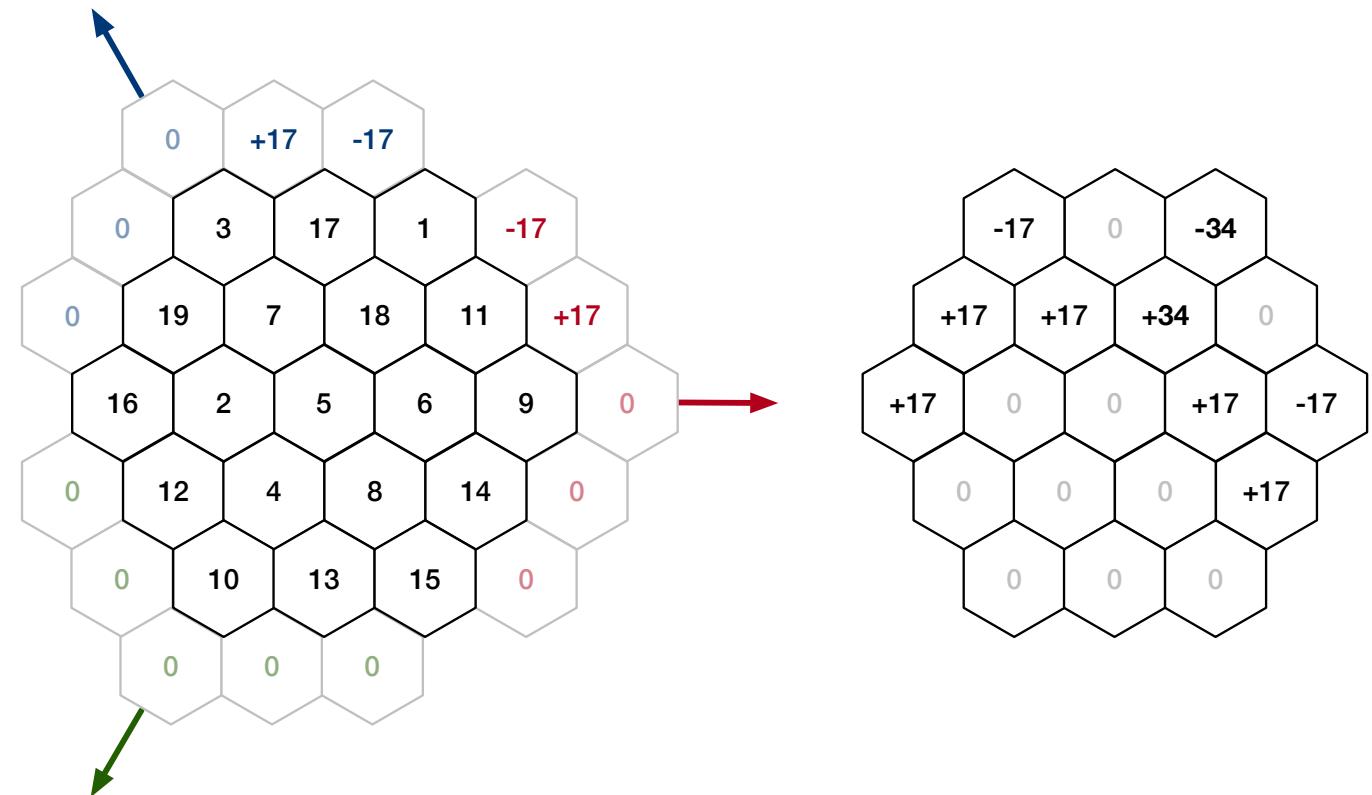
# Improvement #2

## Issue:

- ◆ Fitness function only evaluates equations, not tokens.
- ◆ Humans would

## Solution:

Add token-based fitness function.



$$|+17| + |-34| + |+17| + |+17| + |+17| + |+34| + |+17| + |+17| + |-17| + |+17| = 204$$

# Improvement #3

## **Issue:**

- ◆ Single mutation swap operation converges too slow.
- ◆ Multiple swap operations can "overshoot".

## **Solution:**

Use random number of swaps.

# Improvement #4

## **Issue:**

Doesn't converge all the time.

## **Solution:**

Use larger population (now 10k).

# Improvement #5

## **Issue:**

It's slow.

## **Solution:**

Iterating over **HashSet** is slow, don't do that.

# Improvement #5

## Issue:

It's still slow.

## Solution:

Use `[u8; 19]` instead of `Vec<u8>` for candidates.

# Improvement #6

## Issue:

Still, could be faster.

## Solution:

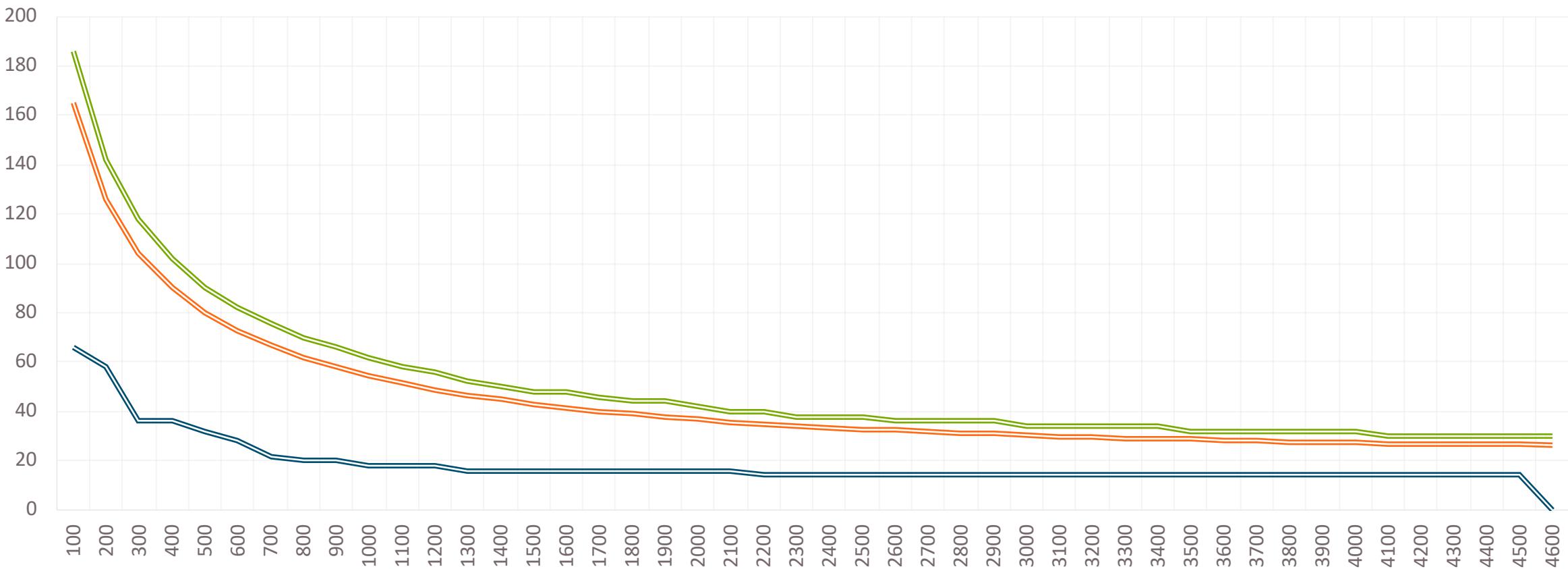
Tune compilation in [Cargo.toml](#):

```
[profile.release]
debug = true
panic = "abort"
lto = true
codegen-units = 1
```

# Convergence

## METRICS OVER ROUNDS

min avg max



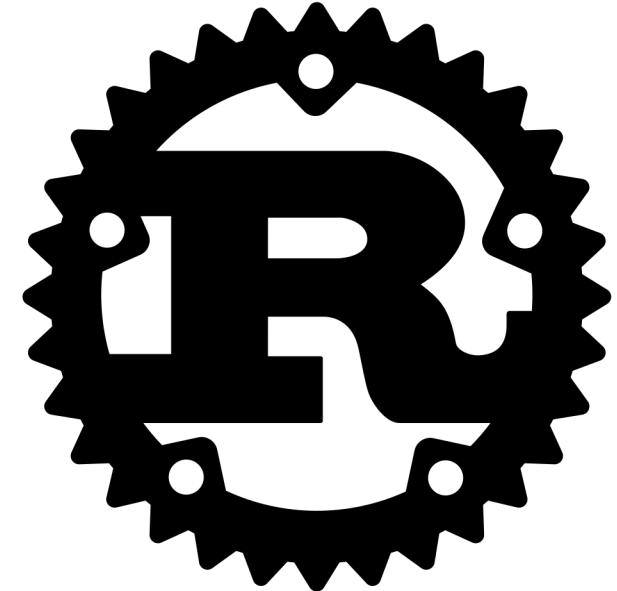
# How Rust Helps

## Compact

- ◆ 142 lines, well formatted
- ◆ No weird performance hacks required

## Compiler

It compiles → it runs.



## Performance

Benchmark #1: cargo run --release

Time (mean  $\pm \sigma$ ): 48.387 s  $\pm$  23.373 s [User: 41.296 s, System: 0.858 s]

Range (min ... max): 19.084 s ... 104.775 s 10 runs

# Potential Extensions

## Performance

- ◆ Parallel Solution
- ◆ Pre-calculate fitness function

## Mutation

- ◆ Better swap count (e.g. “simulated annealing”)
- ◆ Intelligent swap (e.g. use token-based rating)
- ◆ Crossover
- ◆ Normalize rotation and mirroring

# Appendix

# Other Applications

- ◆ Other Puzzles like Sudoku

- ◆ Conference Scheduling:

<https://medium.com/@filiph/a-genetic-algorithm-scheduled-our-developer-conference-heres-what-i-learnt-eac0069709f5>

- ◆ Training Neural Networks:

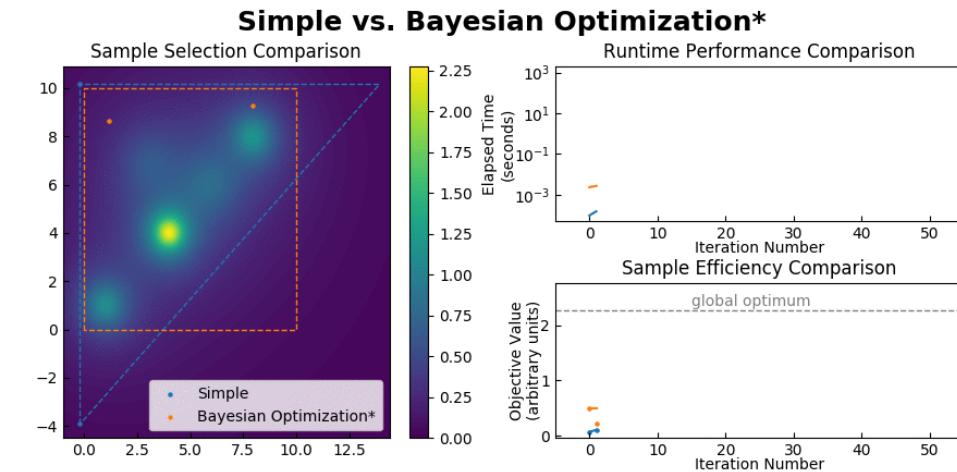
<https://openai.com/blog/evolution-strategies/>

# Frameworks #1

## Simple(x) Global Optimization

... to deal with a large number of sample and high dimension efficiently.

<https://github.com/nestordemeure/Simplers>



source: <https://github.com/chrisstroemel/Simple>

## argmin{}

Mathematical optimization in pure Rust  
<http://argmin-rs.org/>

```
// First, create a struct for your problem
#[derive(Clone, Default, Serialize, Deserialize)]
struct Rosenbrock {
    a: f64,
    b: f64,
}

// Implement `ArgminOp` for `Rosenbrock`
impl ArgminOp for Rosenbrock {
    /// Type of the parameter vector
    type Param = Vec<f64>;
    /// Type of the return value computed by the cost function
    type Output = f64;
    /// Type of the Hessian. Can be `()` if not needed.
    type Hessian = Vec<Vec<f64>>;

    /// Apply the cost function to a parameter `p`
    fn apply(&self, p: &Self::Param) -> Result<Self::Output, Error> {
        Ok(rosenbrock_2d(p, self.a, self.b))
    }

    /// Compute the gradient at parameter `p`.
    fn gradient(&self, p: &Self::Param) -> Result<Self::Param, Error> {
        Ok(rosenbrock_2d_derivative(p, self.a, self.b))
    }

    /// Compute the Hessian at parameter `p`.
    fn hessian(&self, p: &Self::Param) -> Result<Self::Hessian, Error> {
        let t = rosenbrock_2d_hessian(p, self.a, self.b);
        Ok(vec![vec![t[0], t[1]], vec![t[2], t[3]]])
    }
}
```

# Frameworks #2

## genevo

... provides building blocks to run simulations of optimization and search problems using genetic algorithms

<https://github.com/innoave/genevo>

```
// The fitness function for `Selection`
impl<'a> FitnessFunction<Selection, i64> for &'a Problem {
    fn fitness_of(&self, selection: &Selection) -> i64 {
        ...
    }

    fn average(&self, values: &[i64]) -> i64 {
        (values.iter().sum::<i64>() as f32 / values.len() as f32 + 0.5).floor() as i64
    }

    fn highest_possible_fitness(&self) -> i64 {
        self.highest_possible_fitness
    }

    fn lowest_possible_fitness(&self) -> i64 {
        0
    }
}
```

## oxigen

a parallel genetic algorithm library implemented in Rust

<https://github.com/Martin1887/oxigen>

```
impl Genotype<u8> for QueensBoard {
    type ProblemSize = u8;

    fn iter(&self) -> std::slice::Iter<u8> { ... }
    fn into_iter(self) -> std::vec::IntoIter<u8> { ... }
    fn from_iter<I: Iterator<Item = u8>>(&mut self, genes: I) { ... }

    fn generate(size: &Self::ProblemSize) -> Self { ... }

    fn fitness(&self) -> f64 { ... }

    fn mutate(&mut self, rgen: &mut SmallRng, index: usize) { ... }

    fn is_solution(&self, fitness: f64) -> bool { ... }
}
```

# Source Code

[https://github.com/crepererum/aristotles\\_number\\_puzzle\\_genetic](https://github.com/crepererum/aristotles_number_puzzle_genetic)

# Literature

- ◆ “Essentials of Metaheuristics”

2013

Sean Luke

<https://cs.gmu.edu/~sean/book/metaheuristics/>

- ◆ “Learning Bayesian Network Structures by searching for the best ordering with genetic algorithms”

1996

Pedro Larrafiaga, Cindy M. H. Kuijpers, Roberto H. Murga, Yosu Yurramendi

<https://www.cse.unr.edu/~sjose/Papers%20Referenced/Learning%20Bayesian%20Network%20Structures%20by%20Searching%20for%20the%20Best%20Ordering%20with%20Genetic%20Algorithm.pdf>